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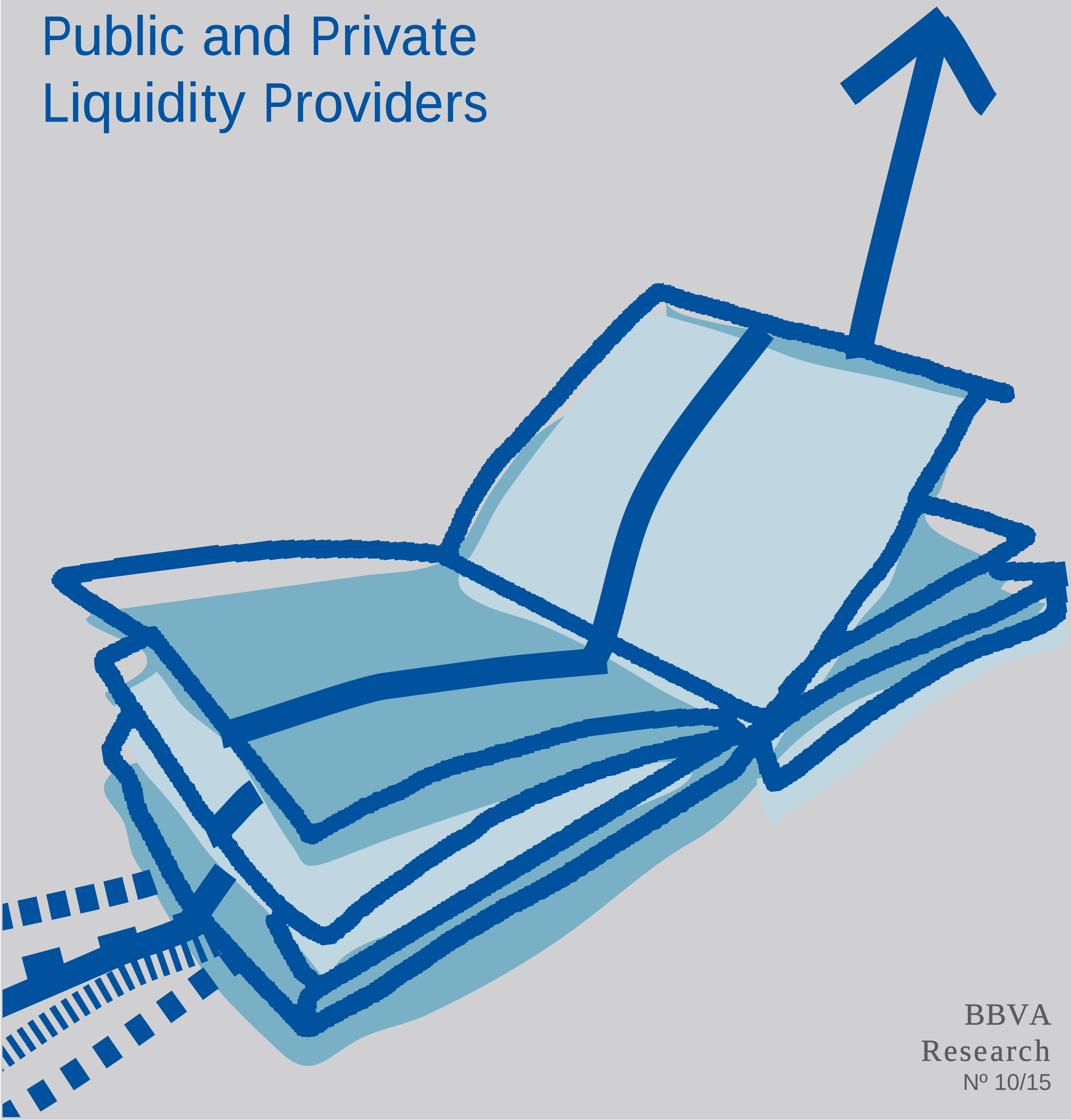
Public and Private Liquidity Providers



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Public and Private Liquidity Providers

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Abstract

The goal of this paper is to explore the benefits of having a private liquidity provider and the conditions under which this lender provides liquidity, when a public liquidity provider is also present. The model proposed incorporates an endogenous interbank lending market so that the decision of a bank to seek liquidity in the interbank market or to turn to the private or the public lender is also endogenous. This framework permits the derivation of conclusions on the size of the private lender, interbank lending conditions and optimal policy for liquidity provision.

1 Introduction

A key responsibility of central banks is provision of liquidity to banks in periods of financial stress. However, there are privately-owned companies that also have that objective. In the United States, the Congress created the Federal Home Loan Banks (FHLBanks) as a system of government-sponsored enterprises, federally chartered but privately capitalized and independently managed, and gave them the mission, among other responsibilities, to serve as a reliable source of liquidity for their membership. The FHLBanks pursued their mission and provided liquidity to their member banks during the crisis. They had a very important role in liquidity provision after the bankruptcy of Lehman Brothers as other sources of liquidity such as the financial markets and the interbank market restrained. Their loans reached its maximum in the third quarter of 2008 (see Figure 1).

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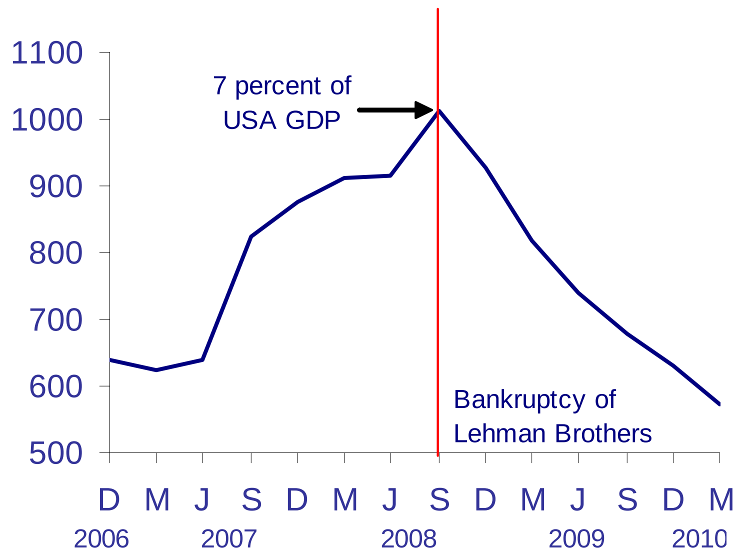

Figure 1. FHLBanks: Advances to Commercial Banks
(billions of dollars).

Similar institutions exist also in other countries. For example, in Germany, the banks associations created the Liquiditäts-Konsortialbank or Liko bank to provide liquidity to its members (70% owned by individual members of the banking system and 30% by the Bundesbank). The purpose of this paper is to analyze the social benefits of the existence of these institutions and their interaction with the public lender of last resort.

Although the literature on the lender of last resort (LOLR) is extensive (Freixas et al., 2000 for a recent survey), to our knowledge, papers that analyze the interaction between private and public liquidity providers are scarce. However, some efforts have been made. For example, Herrala (2001) investigates whether voluntary schemes such as public lender of last resort, a mutual clearing house, and a profit-maximizing private LOLR are, in terms of social utility, as good as a compulsory LOLR scheme. Our paper, in contrast, permits the interaction between a bank that requires liquidity and the different potential providers. In order to analyze this interaction we construct a model where the decision of a bank to seek liquidity in the interbank market or to turn to a private or a public lender is endogenous. This framework allows us to derive conclusions on the size of the private lender, interbank lending conditions and optimal policy for liquidity provision. Research about optimal LOLR institutional design has been done (see Repullo, 2000). However, it has considered only public sector institutions: the central bank and the deposit insurance corporation.

The results derived from this paper are the following. First, the private liquidity provider supplies resources to banks only when the amount lent is high enough. This could explain why the Federal Home Loan Banks hold total assets of around 6 per cent of US GDP. This result suggests a relevant corollary: a public liquidity provider is necessary for small liquidity provisions. Second, the less correlated the shocks across banks are, the more difficult interbank lending is, which makes a lender of last resort institution even more important. When the shocks are correlated a bank is more willing to lend to other bank as it can obtain some profit even if a bad shock hits both banks as this model permits the lending bank to keep some remnant from the project financed. This remnant can be seen as a collateral that may cover a share of the loan. Third, in the case that the amount of liquidity required is high and there is a stigma or reputational cost for a bank that obtains liquidity from the public lender, the existence of a private liquidity provider is welfare-improving. Notwithstanding, a public liquidity provider should exist as it improves welfare when the liquidity needs and the reputation costs are small. This result supports the need for a public lender of last resort.

The paper is organized as follows. Section 2 presents the model and social welfare analysis for different liquidity provision arrangements. Section 3 provides some concluding remarks.

2 Model

This model assumes an economy that is formed by two banks that act as local monopolies and face a continuum of consumers of size one. There is a public deposit insurance corporation that may also act as a liquidity provider and a private liquidity provider.

The economy has three periods. At period $t = 0$ each bank raises one unit of deposits. For simplicity we assume that consumers are unable to store consumption goods, and hence the bank provides a valuable service since consumers prefer to consume at $t = 2$. At $t = 1$ consumers may receive a shock that forces them to withdraw their deposits. The fraction of deposits withdrawn early is a random variable $0 \leq v \leq 1$ with distribution function $F(v)$ and density $f(v) < z$.

The banks are endowed with a constant returns to scale technology that yields a different return depending on the state of the world. In state high, h, it yields R_h whereas in state low, l, it yields R_l. Let $R_l < 1 < R_h$. The probability of state h is p and of state l is the complementary probability. We also have that $\overline{R} = pR_h + (1-p)R_l > 1$, and therefore the project makes sense since it has positive expected value. The technology requires two periods to mature. Thus, it delivers the returns in period $t = 2$. Early liquidation is possible, but with a cost. Let $L \leq 1$ be the total resources obtained after liquidation. The difference $1 - L$ represents the cost of early liquidation.

For explanatory purposes consider first no liquidity from outside sources. Hence we have that at $t = 0$ consumers deposit 1 at the bank, at $t = 1$ the value of v is realized and the bank has to liquidate part of the project, if it is enough to pay v the bank continues to next period, otherwise the bank fails and the depositors get the liquidation value L. Therefore, we have that the bank is able to continue for the next period if $v < \widehat{v}$, with $\widehat{v} = L$. If the bank survives in $t = 2$ the state of the world unfolds and the return is known. If the return of the project is enough to pay depositors, the rest is for the bank, if not, $1 - v$ depositors get whatever is left. In state l, patient depositors are not fully paid. Figure 2 summarizes the basic timing of events.

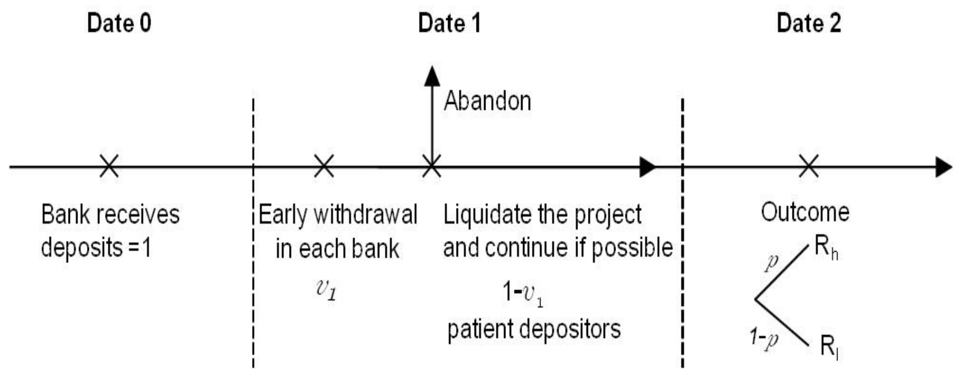

Figure 2. Basic timing of events.

For explanatory purposes, consider first the problem when there is only a bank. This is, there is no interbank lending market. We will expand the analysis to incorporate the role of the interbank lending market in relation to other potential sources of liquidity in the next sub-section.

Let α be the proportion of deposits required to be liquidated in order to pay the withdrawal at $t = 1$. So we have that αL must equal v, so $\alpha = \frac{v}{L} \geq v$.The expected profits of bank 1 in the non-run equilibrium are:

$$E(\pi_{B_1}) = \int_0^{v_1^*} p \left[\left(1 - \frac{v_1}{L_1} \right) R_h - (1 - v_1) \right] f(v_1) dv_1$$

where the subscript 1 refers to the bank and $v_1^* = \frac{(R_h - 1)L_1}{R_h - L_1}$. The value v_1^* represents the benchmark of v_1 that permits the bank to pay the patient consumer. For $v_1 > v_1^*$ the return of the project is not enough to pay the patient depositors. Note that $v_1^* < \widehat{v}_1 = L_1$. Also note that in state l returns are never enough for paying patient depositors. The bank does not have negative profits, and hence does not absorb losses to pay patient consumers, due to limited liability .

The deposit insurance institution (DI) could discard the run equilibria. The expected losses of the deposit insurance fund would be:

$$E(\pi_{DI} \quad | \quad {}_{DI\ insure,\ E(\pi_{B_1})}) = \int_0^{v_1^*} (1 - p) \left[(1 - \frac{v_1}{L_1}) R_l - (1 - v_1) \right] f(v_1) dv_1$$
$$+ \int_{v_1^*}^{\widehat{v_1}} \left[(1 - \frac{v_1}{L_1}) \overline{R} - (1 - v_1) \right] f(v_1) dv_1 + \int_{\widehat{v_1}}^{1} (L_1 - v_1) f(v_1) dv_1 \quad (1)$$

Notice that where there is a DI the consumers lose nothing and they do not have any incentive to withdraw if they do not receive a shock.

Additionally, the deposit insurance may lend v_1 to the bank in order to avoid inefficient liquidation of the project. Assuming a zero interest rate for its lending, the expected loss of DI when it lends to the bank is

$$E(\pi_{DI} \mid_{DI\ lend,\ E(\pi_{B_1})}) = (1-p)(R_l - 1). \tag{2}$$

The expected profits of bank 1 when it obtains liquidity from DI are

$$E(\pi_{B1}) = p(R_h - 1) - c, \tag{3}$$

where c is a non-pecuniary cost that has to be paid in case a bank receives a loan from DI. This cost can be seen as a reputation or stigma cost. Jenkinson, N. (2009) considers that this stigma implies that some central bank liquidity facilities are ineffective.[1]

Define $\overline{c} = p(R_h - 1)$, if $c = \overline{c}$ bank 1 finds optimal to borrow from DI only if $v_1 > v_1^*$ and there is utility from finishing the project. Now for every $c < \overline{c}$ we have a $v_1^c = \frac{cL}{p(R_h - L)}$ such that bank 1 borrows only if $v_1 > v_1^c$. The parameter v_1^c is the minimum value of v_1 that permits the bank to make more profits by obtaining credit from the DI than by liquidating the project. This implies that bank 1 asks for a loan only if the size of the withdrawal shock is high enough, otherwise it prefers to under-invest, increasing the cost of the DI in the l state:[2]

$$\begin{aligned} E(\pi_{DI} \quad \mid \quad_{DI\ lend,\ E(\pi_{B_1})}) &= \int_0^{v_1^c} (1-p)\left[(1-\frac{v_1}{L_1})R_l - (1-v_1)\right] f(v_1)dv_1 + \\ &+ \int_{v_1^c}^1 (1-p)(R_l - 1) f(v_1)dv_1 \\ &= (1-p)(R_l - 1) - \int_0^{v_1^c} \frac{(1-p)v_1}{L_1} R_l dv_1. \end{aligned}$$

Allow a private liquidity provider (PL) to lend v_1 to the bank. The expected profit function of the PL is:

$$E(\pi_{PL}) = p(R_h - (1 - v_1)) + (1-p)[R_l - (1-v_1)]^+ - v_1 \tag{4}$$

The factor $(1 - v_1)$ represents the share of deposits that have to be given back to the depositors. This result assumes that the lender expropriates all profits from the bank. This assumption does not seem to be highly unrealistic, as the liquidity provision could be seen as a monopolistic activity. In addition,

[1] Nigel Jenkinson was Executive Director of the Financial Stability Area in the Bank of England.

[2] Notice that the public lender always finds it optimal to supply liquidity because in the case of bankruptcy it would bear the cost of the innefficiency created by the early liquidation.

this assumption only alters the distribution of profits among agents. As we are analyzing the effect on social welfare of the introduction of a private liquidity provider, a different distribution of profits among agents does not alter the results.

Notice that there exists a small v_1 that makes $E(\pi_{PL}) < 0$. This happens for $v_1 < \widetilde{v_1}$, where $\widetilde{v_1} = 1 - R_l$ (see proof in the annex). This implies that the PL provides liquidity only when the amount to be lent (withdrawal shock) is high enough (higher than threshold $\widetilde{v_1}$). The intuition behind is that the expected benefits to PL are enough to pay its investment only when the patient depositors, which are paid first, are few, and therefore it can expropriate enough resources. As a bank failure has detrimental effects on the functioning of the economy (see Goodhart, 1995), the previous result implies that a public lender should exist and act when the PL is not willing to do so (assuming the absence of an interbank market). Bank 1 borrows from the PL only if $v_1 > v_1^*$ and $c > \overline{c}$.

Figure 3 shows under which conditions PL and DI provide liquidity and bank 1 chooses to liquidate its project.[3] When the cost for the bank of receiving a loan from DI is high enough (i.e. $c > \overline{c}$), bank 1 chooses to liquidate the project when the early deposits withdrawal is small enough to obtain profits ($v_1 < L$). When the early withdrawal does not permit bank 1 to obtain profits, it would be indifferent between obtaining liquidity from PL or liquidating the project. As bank 1 can survive by borrowing from PL we assume that it prefers to borrow and survive than to liquidate the project and abandon the market. In addition, obtaining liquidity from PL is a Pareto improvement as bank 1 remains as before, PL obtains profits and patient consumers get back their deposits. The dark area indicates when the PL provides liquidity. The results are supported by Bech et al. (2007) who find that the use of FHLBank advances by financial institutions in contrast to borrowing from the discount window, is the result of a lack of stigma, among other factors.

Bank 1 chooses to obtain a loan from DI rather than to liquidate the project when the deposits withdrawal is high enough to obtain profits after facing the cost of receiving the loan (i.e. $v_1 > v_1^c$). The area where this occurs is indicated by the letters DI. For values $v_1 > L$, obtaining liquidity from DI is a Pareto improvement as patient consumers obtain all their deposits, the whole project matures, and therefore the output is higher.

[3]For illustration pourposes in this figure we assume that $L_1 > (1 - R_l)$.

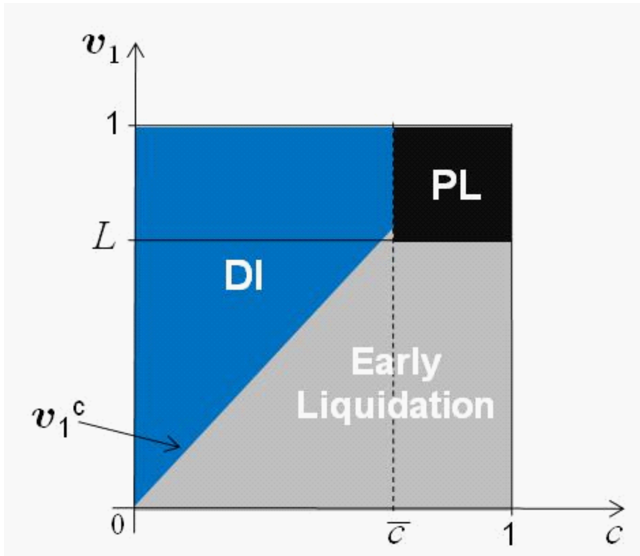

Figure 3. Areas of liquidity provision.

2.1 Interbank Lending Market

In order to expand the analysis, we now consider bank 2, B_2, and the role of the interbank lending market in relation to other potential liquidity providers. For simplicity, a scale problem is assumed so that banks can receive no more than 1 in deposits. In this model, assume that bank 2 is efficient, meaning it can fully liquidate its assets at time 2 without cost, $L_2 = 1$. The expected profits of bank 2 at $t = 1$ are:

$$E(\pi_{B_2}) = p\left[(1 - v_2)R_h - (1 - v_2)\right] = p(R_h - 1)(1 - v_2) \quad (5)$$

Bank 2 asks for a loan to the DI if and only if $c < v_2 p(R_h - 1)$. There exists a $v_2^* = \frac{c}{p(R_h-1)}$ such that B_2 finds it attractive to borrow from the DI. In such a case $E(\pi_2) = p(R_h - 1) - c$. But notice that B_2 only borrows in order to obtain higher profits, so we can assume that the DI refuses to lend in that case. In the case of the private liquidity provision, B_2 never finds it optimal to borrow from the PL because it would lose all of its profits. Notice also that B_2 cannot borrow from the DI and use these resources to lend to B_1 in order to obtain profits using public funds.

The expected losses for the insurance fund from this bank are:

$$E(\pi_{DI} \mid_{DI\ insure, E(\pi_{B_2})}) = (1-p)\left[(1 - v_2)R_l - (1 - v_2)\right] = (1-p)(R_l-1)(1-v_2) \quad (6)$$

Given that bank 2 is efficient it could lend to bank 1 at $t = 1$. Ex-ante expectations about the size of withdrawal shocks or loans are not important given that no decision has to be made at $t = 0$. The decision to lend and borrow will be made once v_1 and v_2 are known. The timing of events when bank 1 has access to liquidity sources is described in Figure 4.

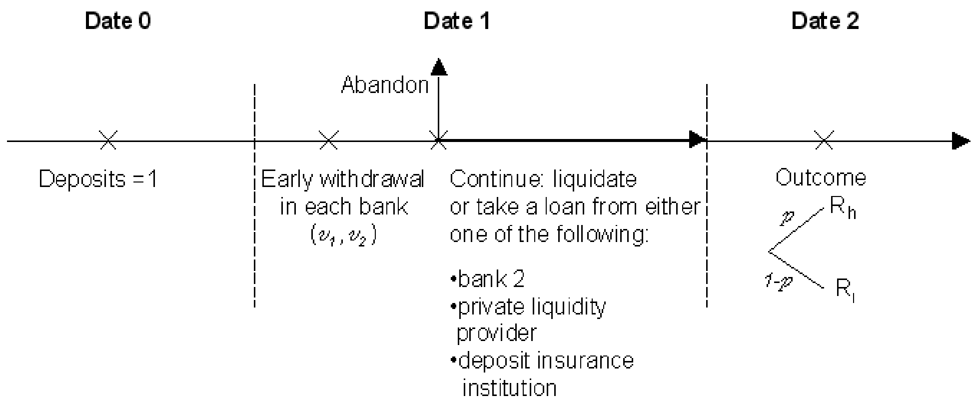

Figure 4. Timing of events.

The possibility of an interbank market enriches the decision space with four possible outcomes at $t = 2$. Let y be the amount bank 2 lends to bank 1 and R be the interest rate on the loan, $R = 1 + r$. For simplicity, in our framework the surplus bank providing liquidity does not have market power in the interbank market, and thus, it does not strategically under-provide lending as in Acharya, Gromb and Yorulmazer (2008).

The decision for bank 1 on whether to take funds from bank 2 depends on the values of v_1 and c, given that both the public and the private liquidity provider exist in this world. Hence, the decision for bank 1 could be described as:

$$\text{Borrow if } E(\pi_{B_1}|y) \geq \begin{cases} 0 & \text{if} \quad c > \bar{c} \text{ and } v_1 > v_1^* \\ p(R_h - 1) - c & \text{if} \quad c < \bar{c} \text{ and } v_1^* \geq v_1 > v_1^c \\ p\left[(1 - \frac{v_1}{L_1})R_h - (1 - v_1)\right] & \text{if} \quad c < \bar{c} \text{ and } v_1 \leq v_1^c. \end{cases}$$

In the last two cases bank 1 borrows from bank 2 if $R \leq \frac{R_h}{L}$. This is when the price bank 1 has to pay to bank 2 for the liquidity is smaller than the potential benefit of the project adjusted by the liquidation efficiency. In the first case of the inequality, the set of values where bank 1 borrows from bank 2 exist but are hard to achieve given that v_1 is large, bank 2 can only lend from its own funds and $R < \frac{R_h}{L}$. Although it is possible that the interbank market exists in such an environment, the probability is small. Notice that the decision of bank 1 does not depend on the state of the world that affects bank 2. However, the way events unfold in $t = 2$ is relevant for bank 2.

Let the pair (a, b) represent the state of nature at $t = 2$, the first coordinate corresponds to the state bank 1 faces and the second to the state of bank 2.

Profits for bank 2 when it lends to bank 1 are:

$$\pi_{B_2} = \begin{cases} (1-v_2)R_h - (1-v_2) + y(R-R_h) & \text{if} \quad (h,h) \\ (1-v_2)R_h - (1-v_2) + t - yR_h & \text{if} \quad (l,h) \\ (1-v_2)R_l - (1-v_2) + y(R-R_l) & \text{if} \quad (h,l) \\ (1-v_2)R_l - (1-v_2) + t - yR_l & \text{if} \quad (l,l) \end{cases} \tag{7}$$

with $t = \left[(1-\frac{v_1-y}{L})R_l - (1-v_1)\right]^+$ being the remnant from bank 1 in state l. In order to calculate the $E(\pi_{B_2})$ the probability of occurrence of each state for each bank is needed. Since we have only two states, (h,l), a Bernoulli distribution is appropriate. It is assumed that the states h and l for each bank are correlated with parameter θ. The probability matrix is then given by:

$$\begin{bmatrix} P(h,h) & P(h,l) \\ P(l,h) & P(l,l) \end{bmatrix} = \begin{bmatrix} p^2 + \theta p(1-p) & p(1-p)(1-\theta) \\ p(1-p)(1-\theta) & (1-p)^2 + \theta p(1-p) \end{bmatrix} \tag{8}$$

The expected profits are subject to limit liability, so depending on the size of R and y the expected profit in a state may be zero. The expected profits function for bank 2 is:

$$E(\pi_{B_2}|y) = \begin{cases} p\left[(1-v_2)(R_h-1) - yR_h\right] + dyR + et & \text{if} \quad y < \widetilde{y} \\ p\left[(1-v_2)(R_h-1) + y(R-R_h)\right] + e\left[t + (1-v_2)(R_l-1) - yR_l\right] & \text{if} \quad y \in [\widetilde{y}, \widehat{y}] \\ d\left[(1-v_2)(R_h-1) - yR_h\right] + e\left[(1-v_2)(R_l-1) - yR_l)\right] + pyR & \text{if} \quad y > \widehat{y}. \end{cases} \tag{9}$$

Where $\widetilde{y} = (1-v_2)\left(1-\frac{1}{R_h}\right)$ and $\widehat{y} = (1-v_2)\left(\frac{1-R_l}{R-R_l}\right)$ represent the thresholds of y that permit bank 2 to make positive profits and therefore be willing to give a loan. Let $d = p^2 + \theta p(1-p)$ and $e = p(1-p)(1-\theta)$.

When R is large enough the $E(\pi_{B_2}|y)$ is lowest when $y < \widetilde{y}$.

The expected profits of bank 1 when it receives a loan from bank 2 are:

$$E(\pi_{B_1} \mid y) = \begin{cases} p[(1-\frac{v_1-y}{L})R_h - (1-v_1) - yR] & \text{if} \quad (h,h) \text{ or } (h,\ l) \\ 0 \text{ otherwise.} \end{cases} \tag{10}$$

The probability matrix allows us to obtain a scalar of the previous function.

For bank 2 it is optimal to give a loan if $E(\pi_{B_2}|y; y < \widetilde{y}) > E(\pi_{B_2}|y = 0)$. This occurs when

$$R > \frac{R_h}{p + \theta(1-p)}.$$

When $\theta \to -1$ it is less likely that the inequality holds (as long as $p > \frac{1}{2}$). That is, the less correlated the shocks are, the more difficult it is for bank 2 to give a loan. Although this is not an obvious finding, the intuition that lies behind is that when the shocks are correlated, bank 2 is more willing to lend to bank 1 in the high state as it recovers its loan and makes profits, and in the low state instead of having only losses it keeps the return from the loan to bank 1.

that lies behind is the following. There are two states, high and low. When the shocks are correlated, in the high state bank 2 lends to bank 1 as it recovers its loan and makes profits. In the low state, bank 2 instead of having only losses it keeps the return from the loan to bank 1.

When bank 2 does not give a loan to bank 1 (e.g. $R > \frac{R_h}{p+\theta(1-p)}$ does not hold) bank 1 has to borrow from the private lender or the deposit insurance institution.

Bank 1 prefers to obtain a loan from bank 2 rather than to liquidate when $E(\pi_{B_1}|y) > E(\pi_{B_1}|_{Liquidate})$. This occurs as long as

$$R \leq \frac{R_h}{L}.$$

This is, the payoff for bank 1 in the high state, adjusted for its liquidation efficiency, is higher than the cost of a loan from bank 2.

Bank 1 prefers to obtain a loan from bank 2 rather than from DI when $E(\pi_{B_1}|y) > E(\pi_{B_1}|_{DI\ lend})$. This is when

$$v_1 \leq v_1^{DI} = \frac{y(R_h - LR)}{R_h - L} + \frac{L}{(R_h - L)}\frac{c}{p}.$$

When the loan required is high enough, $v_1 > v_1^{DI}$, bank 1 prefers not to borrow from bank 2, but rather to borrow from DI as the return of the project is not enough to pay bank 2 yR and obtain profits higher than those obtained by borrowing from DI.

Bank 1 prefers to obtain a loan from bank 2 rather than from PL when $E(\pi_{B_1}|y) > E(\pi_{B_1}|_{PL}\ )$. This is when

$$v_1 \leq v_1^{PI} = \frac{y(R_h - LR)}{R_h - L} + \frac{L(R_h - 1)}{(R_h - L)}.$$

When the loan required by bank 1 from bank 2 is small enough,$v_1 \leq v_1^{PI}$, the benefits of the project are enough to pay back bank 2 and make positive profits.

The next figure shows the areas where each agent provides liquidity to bank 1.

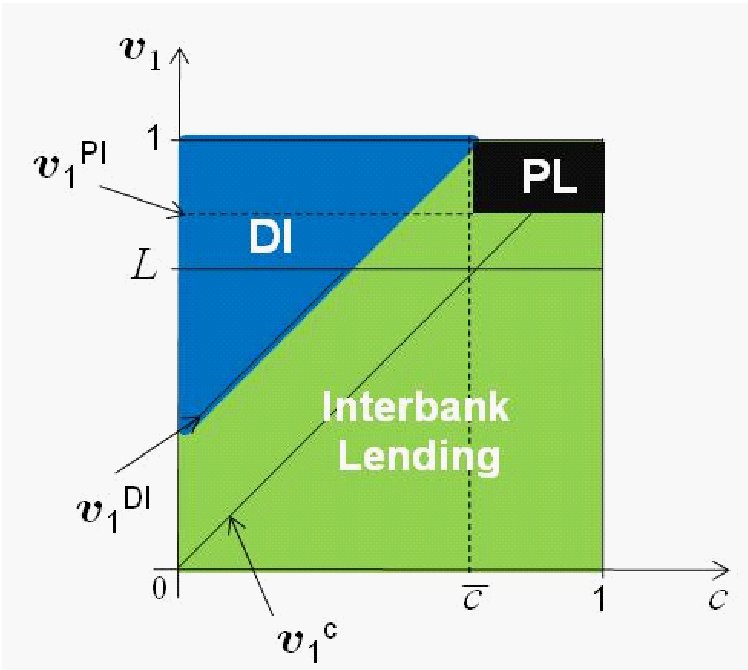

Figure 5. Areas of liquidity provision under interbank lending.

2.2 *Social Welfare Analysis*

The social welfare is defined as the sum of benefits/costs of bank 1, bank 2, the private liquidity provider and the deposit insurance institution, and is calculated for each one of the liquidity provision outcomes. For illustration purposes we assume a uniform density function, $f(v) = 1$, $0 < v < 1$.

First, consider that bank 2 does not provide liquidity to bank 1. (e.g. $R > \frac{R_h}{p+\theta(1-p)}$ does not hold). In this case bank 1 can obtain liquidity from the PL, the DI or can decide to liquidate its project. The social welfare associated to each one of these three outcomes is the following:

Case 1. Only PL provides liquidity (i.e. $c > \overline{c}$ and $v_1 > v_1^*$).

$$W(v) \quad | \quad _{PL} = p(R_h - 1)(1 - v_2) + [p(R_h - (1 - v_1)) + (1 - p)[R_l - (1 - v_1)]^+ - v_1] \\ +(1 - p)(R_l - 1)(1 - v_2) = (\overline{R} - 1)(2 - v_2). \tag{11}$$

Let $a = (\overline{R} - 1)(2 - v_2)$. This represents the total welfare of the economy.

Case 2. Only DI provides liquidity (i.e. $c < \overline{c}$ and $v_1 > v_1^c$)

$$\begin{aligned} W(v) \quad | \quad & _{DI} = p(R_h - 1) - c + p(R_h - 1)(1 - v_2) + (1 - p)(R_l - 1) \\ & +(1 - p)(R_l - 1)(1 - v_2) \\ = \quad & (\overline{R} - 1)(2 - v_2) - c = a - c. \end{aligned} \tag{12}$$

Case 3. Neither PL nor DI provide liquidity.

$$W(v)\mid_{No\ Liquidity} = a + \overline{R}\left(\frac{L}{2} - 1\right) + \frac{1}{2}. \tag{13}$$

The scalar $\overline{R}\left(\frac{L}{2} - 1\right) + \frac{1}{2}$ represents the cost of bank 1's project not being able to mature.

Now, consider the social welfare when there is interbank liquidity.

Case 4. Bank 2 provides liquidity to bank 1.

$$W(v)\mid_{Bank\ 2\ provides\ liquidity} = a - \left[y(\overline{R} - 1 + R(\theta - 1)(p - p^2)) - et\right] \tag{14}$$

The expression $-\left[y(\overline{R} - 1 + R(\theta - 1)(p - p^2)) - et\right]$ characterizes the cost to not permit to the share y of the project of bank 2 to mature plus an expected remnant from bank 1 in state l, in case it exists. The liquidation of a share of the project implies that the output of the economy is below its potential.

Figure 6 shows the pattern of social welfare in each case.

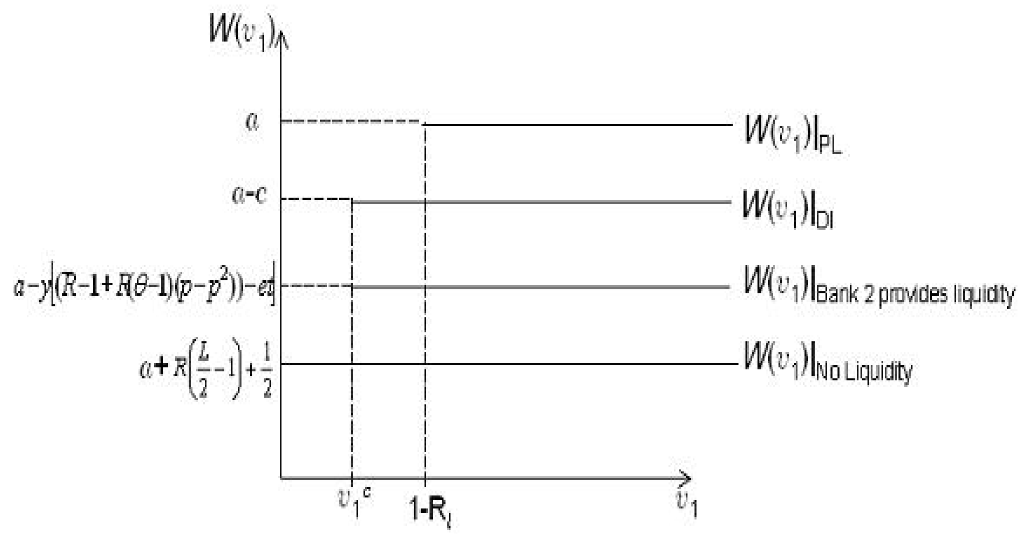

Figure 6. Social welfare.

This figure shows four implications of the model.

1. When $v_1 > 1 - R_l$, it is better that the PL provides liquidity rather than the DI. This occurs due to the reputational cost of obtaining liquidity from the DI.

2. Notice that the lower R_l is, the lower are the values of v for which the PL provides liquidity, as it would not obtain enough profits in the low state.

3. In terms of social welfare, it is better that the DI provides liquidity than the bank 2 does it and than to have no liquidity at all, as long as the non-pecuniary cost, c, is smaller than the cost of the project not being able to mature.

4. When $v_1 < 1 - R_l$ there is a v_1, $v_1 > v_1^c$, where DI provides liquidity and PL does not. This occurs when

$$c < \frac{p(R_h - L)(1 - R_l)}{L}$$

as bank 1 is willing to obtain liquidity from DI when the reputation cost, c, is low enough. This implies that for small shocks, the presence of DI is welfare-improving.

The difference between $W(v) \mid_{No\ Liquidity}$ and $W(v) \mid_{Bank\ 2\ provides\ liquidity}$ depends on the share y of the project of bank 2 that was not able to mature, its return, the cost of the loan from bank 2 and the resources obtained after bank 1 liquidates its project, among others. For illustration purposes, in this graph has been assumed that the cost of bank 1's project is larger than the cost of the share y of the project of bank 2 when they are not being able to mature. This does not seem unrealistic considering that bank 2 is more efficient in liquidating its project.

3 Conclusions

The conclusions of this paper can be summarized as follows:

1. The PL provides liquidity only when the amount lent (withdrawal shock) is high enough, so private liquidity providers are expected to be large. This may explain why the FHLBanks hold total assets that amount to 6 per cent of US GDP.

2. The less correlated the shocks across banks are, the harder it is to rely on interbank lending.

3. When the shock is large enough and the provision of liquidity by DI imposes reputation costs, it is better for PL to provide liquidity than for DI to do it.

4. DI should exist as its presence is welfare-improving when the withdrawal shock and the reputation costs are small.

5. In terms of social welfare, the interbank market is the worst of the liquidity provision options as a bank decides to liquidate a share of its project to provide liquidity to other bank. In contrast, when PL or DI provides liquidity no project has to be liquidated.

Annex

Proof. For PL, there exists a small v_1 that makes $E(\pi_{PL}) < 0$.

The expected profits function of PL is given by $E(\pi_{PL}) = p(R_h - (1 - v_1)) + (1 - p)[R_l - (1 - v_1)]^+ - v_1$. We can identify two cases, where $[R_l - (1 - v_1)]$ is positive and zero, as banks have limited liability.

Case 1. $[R_l - (1 - v_1)] > 0$ or what is the same, $v_1 > \widetilde{v_1}$, where $\widetilde{v_1} = 1 - R_l$. In this case the expected profits function becomes $E(\pi_{PL}) = \overline{R} - 1 > 0$.

Case 2. $[R_l - (1 - v_1)] = 0$ or what is the same, $v_1 < \widetilde{v_1}$, where $\widetilde{v_1} = 1 - R_l$. In this case the expected profits function becomes $E(\pi_{PL}) = p(R_h - (1 - v_1)) - v_1$. Profits, $E(\pi_{PL}) \geq 0$, require $v_1 \leq \overline{v_1} = \frac{p}{1-p}(R_H - 1)$. In order for case 2 to hold we require, $\overline{v_1} < \widetilde{v_1}$.

$$\begin{aligned} \frac{p}{1-p}(R_H - 1) &< 1 - R_l \\ pR_h + (1 - p)R_l &< 1 \\ \overline{R} &< 1 \end{aligned}$$

Parameter value $\overline{R} < 1$ is not reasonable. Hence, $\overline{v_1} > \widetilde{v_1}$ and profits are negative. QED. ■

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